Analysis of laser frequency stability using beat-note measurement

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The frequency of even the most stable 'single frequency' laser is not really constant in time, but fluctuates.

The laser's frequency stability is often described in terms of its spectral linewidth. However, while this term provides a simple figure for comparison of various lasers, it lacks detail about the laser frequency noise which is often critical in laser applications.

For a detailed representation of laser frequency stability we use either the Allan variance of the frequency fluctuations or the frequency noise spectrum. These representations are complimentary and originate respectively from a time-domain description and a Fourier-domain description of the frequency fluctuations. Both the Allan variance and the frequency noise spectrum can be calculated from a beat-note measurement of the frequency fluctuations.

Frequency stability measurement

Beat-note measurement

For a detailed analysis of laser frequency stability, the most common measurement techniques are spectral analysis of the laser transmission through a sensitive interferometer (filter) or a beat-note measurement with a reference laser.

The beat-note measurement is a heterodyne technique where the laser output is mixed with the output of a reference laser and the combined signal is measured using a photo detector. The photo detector is a quadratic detector with a limited bandwidth and will only trace the difference frequency between the two lasers; the so-called beat frequency ν_B (provided the beat frequency is within the detector bandwidth).

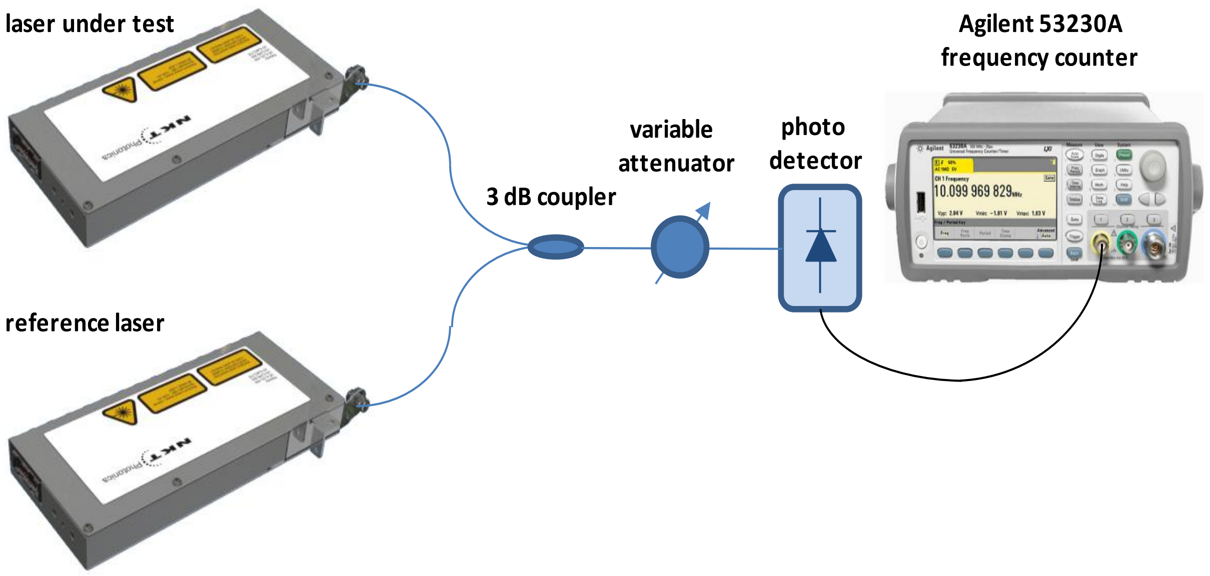

Fig. 1: Beat-note measurement setup

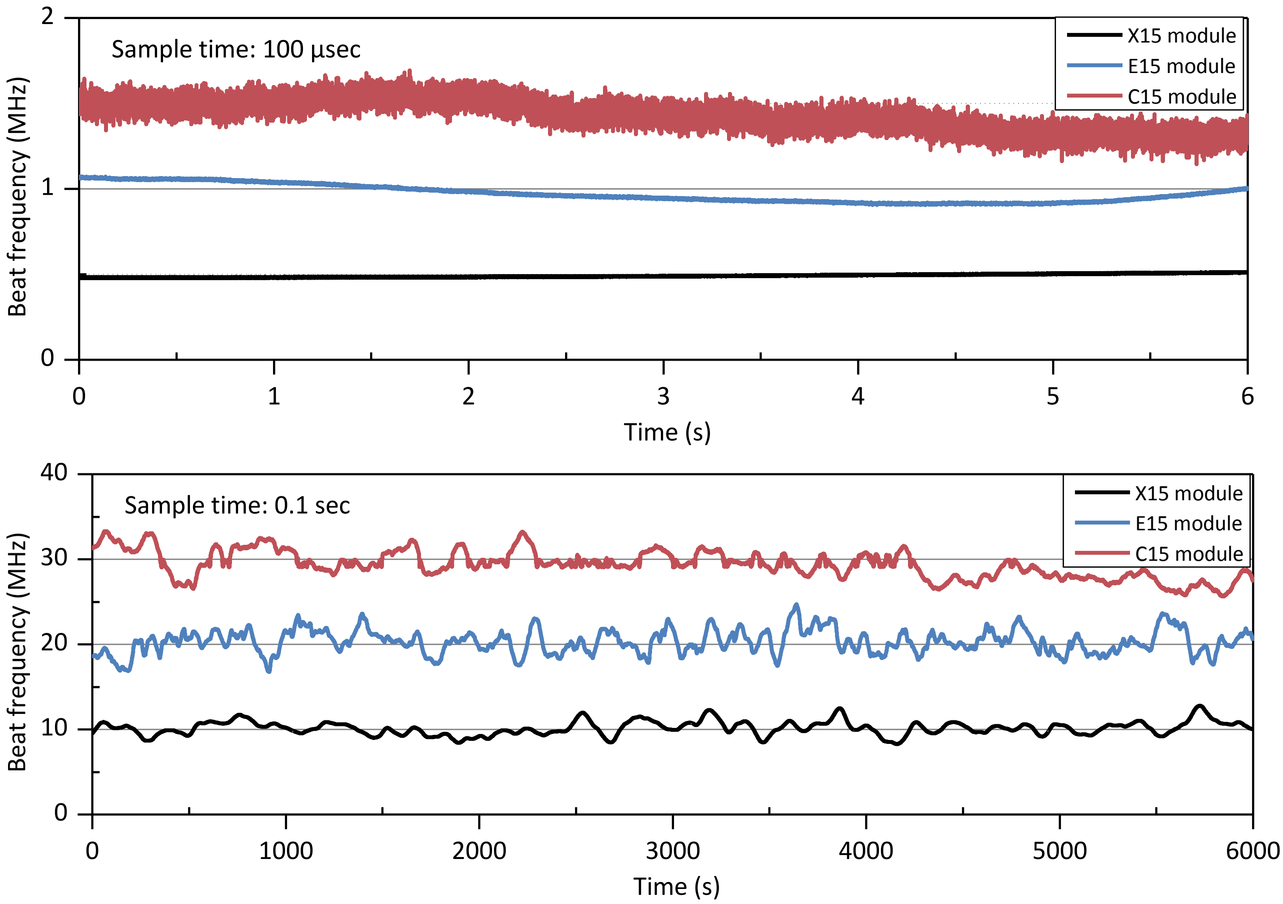

Fig. 2: Short term and long term beat-note measurement between a Koheras X15 AcoustiK reference laser module and various other 1550 nm modules

The beat frequency can be monitored on an electrical spectrum analyzer and recorded by frequency counter.

An experimental setup for measuring and recording the beat frequency is illustrated in fig. 1. The laser output is combined with the reference laser using a fiber coupler and attenuated to a suitable level before entering the photo detector and finally being recorded using a frequency counter.

For a precise measurement of the laser frequency stability, the reference laser should be much more stable than the laser under test. An ideal

The beat-note measurement of laser frequency fluctuations reveals a clear temporal correlation between the measured data. The standard deviation within a subset of the measurement is smaller than the standard deviation of the total measurement interval. To make a meaningful description of the statistical process of the

reference laser source would be a stabilized frequency comb, as this can provide very high frequency stability, and a huge wavelength range which can give a suitable beat frequency for the photo detector. A less ideal reference laser choice is to use a laser similar to the laser under test, though this can make it difficult to estimate the environments influence on the laser stability.

By varying the sample time of the frequency counter, the beat-note setup allows analysis of the frequency fluctuation on a micro-second scale as well a measurement of the long term stability (fig. 2).

frequency fluctuations we need to use a different description than the average frequency and standard deviation. Such a description can be obtained using the Allan variance or the frequency spectrum of the fluctuations.

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Allan variance of frequency fluctuations

For a time domain description of the laser frequency noise we introduce the Allan variance of the instantaneous, normalized frequency deviation, *y(t)*

$$y(t) = \frac{\Delta\nu(t)}{\nu_0}$$

where ν_0 is the optical frequency and $\Delta\nu(t)$ is the instantaneous frequency deviation. The Allan variation is a so-called two-sample variance between consecutive measurements with sample time τ, $\sigma_y^2(\tau)$

$$\sigma_y^2(\tau) = \frac{1}{2}\langle(\bar{y}_2 - \bar{y}_1)^2\rangle$$

where τ is the sample time and the discrete, normalized frequency deviation $\bar{y}_i$

$$\bar{y}_i = \frac{1}{\tau}\int_{t_i}^{t_i+\tau} y(t)\,dt$$

The Allan variation of the frequency fluctuations can be calculated using data from the beat-note measurement by using

$$\bar{y}_i = \frac{\lambda_0}{c\tau}\int_{t_i}^{t_i+\tau} \Delta\nu_B(t)\,dt$$

In the literature, most people use the Allan deviation $\sigma_y(\tau)$, which is the square root of the Allan variance.

If the reference laser in a beat-note measurement is known to be superior in stability compared to the laser under test, then the calculated Allan deviation is a measure of the tested lasers frequency stability. However, if the two lasers is considered to contribute equally to the beat frequency stability then the Allan deviation of the lasers become

$$\sigma_{y,laser\,1}(\tau) = \sigma_{y,laser\,2}(\tau) = \frac{1}{\sqrt{2}}\sigma_{y,tot}(\tau)$$

The above assumption is reasonable if the lasers are identical and the noise sources that contribute to the laser frequency stability can be considered independent. This is the case for many noise sources such as noise introduced by a pump diode or a temperature control. However, some correlation is expected from environmental noise contributions like room temperature variations, vibrations and acoustic noise.
In fig. 3 are some examples of Allan deviation calculated from beat-note experiments between identical type laser modules. The Y10 laser

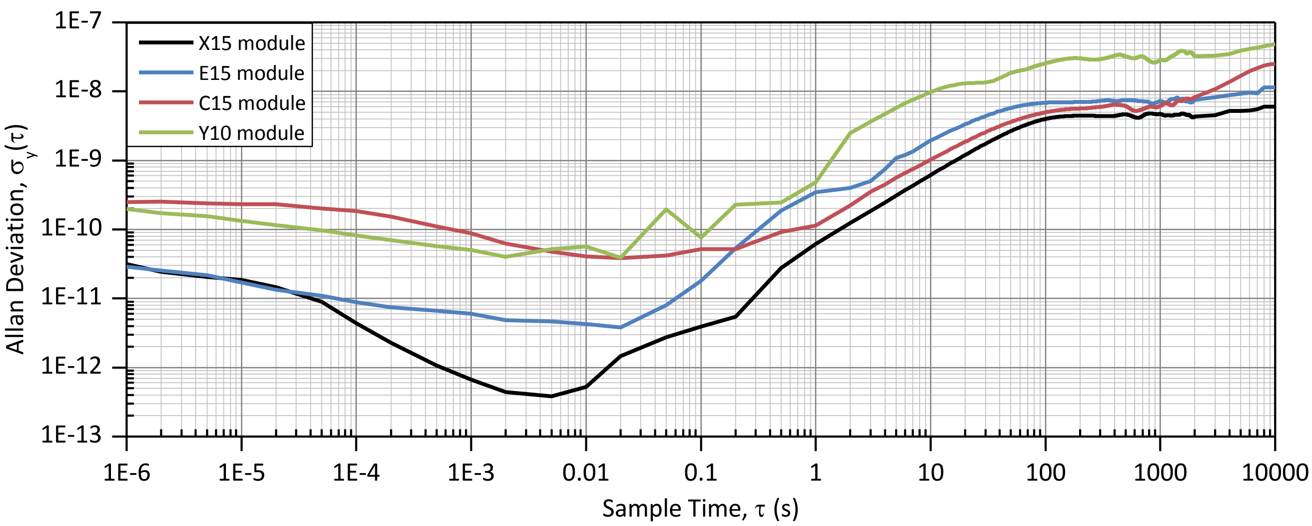

Fig. 3: Allan deviation calculated from beat-note measurement between 2 identical 1064 nm Y10 laser modules and between a X15 AcoustiK reference laser module and various other 1550 nm modules

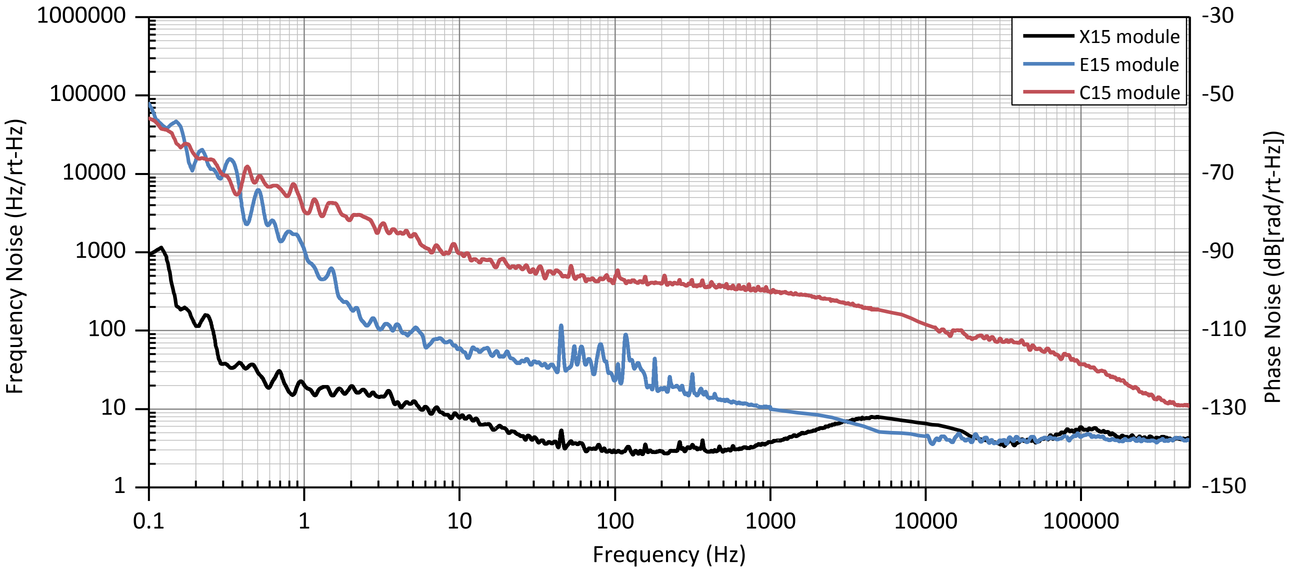

Fig. 4 Frequency noise spectra calculated from beat-note measurement between different fiber laser modules with a X15 AcoustiK module used as reference laser

modules are 1064 nm fiber lasers whereas the other modules are all 1550 nm fiber lasers.

Frequency noise spectrum

For a Fourier-domain description of laser frequency noise we introduce the frequency noise spectrum.

The power spectral density of the frequency spectrum $S_\nu(f)$ has the unit Hz^2/Hz

$$S_\nu(f) = \frac{\left|\mathcal{F}\left(\Delta\nu(t)\right)\right|^2}{BW}$$

where $\Delta\nu(t)$ is the laser frequency fluctuations and *BW* is the measurement bandwidth. We can calculate this using the sampled beat-note measurement data

$$S_\nu\left(k \times \frac{1}{N\tau}\right) = \frac{2\tau}{N}\left|\sum_{n=0}^{N-1}\left(\Delta\nu_{\mathrm{B}}(n) \times \mathrm{win}(n)\right)\mathrm{e}^{-i2\pi nk/N}\right|^2$$

where τ is the sample time, *N* the number of samples, $\Delta\nu_B(n)$ the *n*'th sample of the beat-frequency and win(*n*) is a window function. The window function could be a Hanning function

$$\mathrm{win}_{Hanning}(n) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{N}\right)\right)$$

Often the frequency noise spectrum is represented as the square root of the power density spectrum.

If the reference laser in a beat-note measurement is known to be superior in stability compared to the laser under test, then the calculated frequency noise spectrum is a measure of the tested lasers frequency stability. However, if the two lasers is considered to contribute equally to the beat frequency stability then the Power spectral density of the frequency fluctuation of the lasers become

$$S_{\nu,laser\,1}(f) = S_{\nu,laser\,2}(f) = \frac{1}{2}S_{\nu,tot}(f)$$

In Fig. 4 are some examples of the frequency noise spectrum calculated from beat-note experiments between identical type 1550 nm fiber laser modules. All the measurements are using a X15 AcoustiK module as the reference laser.

Relation between Allan variance and frequency noise spectrum

Both the Allan deviation and the frequency spectrum of the laser frequency fluctuation are useful methods for describing the laser frequency stability. It is possible to calculate the Allan variance from the frequency noise spectrum using the following relation

$$\sigma_y^2(\tau) = 2\int_0^\infty S_y(f)\frac{\sin^4(\pi\tau f)}{(\pi\tau f)^2}df$$

However, the reverse calculation of the frequency noise spectrum is not generally possible.

References / suggested reading

[1] Fritz Riehle, “Frequency Standards, Basics and Applications”, Wiley 2004, chapter 3